(156) N.V. Imp

Exp-3 :- Weingarten Equations :- ~~Show that is~~

State and prove Weigarten eqⁿ.

If L, M, N, E, F, G be fundamental magnitude then W.g eqn

$$H^2 N_1 = (FM - GL) r_1 + (FL - EM) r_2$$

$$H^2 N_2 = (FN - GM) r_1 + (FM - EN) r_2$$

Proof :-

Since N is the unit normal to surface then N_1 and N_2 are $\perp$ to N. so N_1, N_2 lies in tangent plane contain r_1 and r_2 then N_1, N_2 is the linear combination of r_1 and r_2

$$N_1 = a r_1 + b r_2 \quad \text{—— (1)}$$

$$N_2 = c r_1 + d r_2 \quad \text{—— (2)}$$

where a, b, c & d are constant.

Now we have to find a, b, c, d

Taking dot-product of (1) with r_1 and r_2

$$N_1 \cdot r_1 = aE + bF \Rightarrow aE + bF = -L$$

$$N_1 \cdot r_2 = aF + bG \Rightarrow aF + bG = -M$$

$$\Rightarrow \quad aE + bF + L = 0 \quad \text{—— (3)}$$

$$aF + bG + M = 0 \quad \text{—— (4)}$$

$$\frac{a}{FM-GL} = \frac{b}{FL-EM} = \frac{c}{EG-F^2} = H^2$$

$$\Rightarrow a = \frac{FM-GL}{H^2}, \quad b = \frac{FL-EM}{H^2}$$

put in (1), we get

$$N_1 = \frac{FM-GL}{H^2} r_1 + \frac{FL-EM}{H^2} r_2$$

$$\Rightarrow \boxed{H^2 N_1 = (FM-GL) r_1 + (FL-EM) r_2} \quad \text{— (5)}$$

Similarly,

$$\boxed{H^2 N_2 = (FN-GM) r_1 + (FM-EN) r_2}$$

Hence Proved.

(ii) Now Taking dot product of (5) with r_1 & r_2 we get

$$N_2 \cdot r_2 = cF + dG$$

$$N_2 \cdot r_1 = cE + dF \quad \text{— (6)}$$

$$\Rightarrow \left.\begin{aligned} cE + dF &= -M \\ cF + dG &= -N \end{aligned}\right\} \Rightarrow \begin{aligned} cE + dF + M &= 0 \\ cF + dG + N &= 0 \end{aligned} \quad \text{— (7)}$$

$$\frac{c}{FN \mp GM} = \frac{d}{FM - EN} = \frac{+1}{EG - F^2}$$

$$\Rightarrow c = \frac{\pm FN + GM}{H^2}, \quad d = \frac{FM - EN}{H^2}$$

putting these values of c & d in (2)

$$N_2 = \frac{FN - GM}{H^2} r_1 + \frac{FM - EN}{H^2} r_2$$

$$\boxed{H^2 N_2 = (FN - GM) \cdot r_1 + (FM - EN) r_2} \quad \text{— (8)}$$

Hence